Math 45 SSM 2/e 6.2 Factoring trinomials of the form x^2+bx+c

Objectives 1) Factor trinomials of the form x^2+bx+c
2) Factor out GCF from a trinomial, then factor trinomial.

Key observation:
We will factor trinomials (3 terms) in 6.2 and 6.3 but though the problems are similar, they are different enough to require different methods.

Plain 6.2 problem: $x^2+7x+12$

6.2 problem with GCF: $2x^2+14x+24 = 2(x^2+7x+12)$

6.3 problem (no GCF): $2x^2-19x+24$

In 6.2, the leading coefficient is 1: $1x^2+7x+12$
("leading coefficient" means "coefficient of highest degree term")

Review from chapter 5.

Multiply.

① $(x+3)(x+2)$
$= x^2+2x+3x+6$
$= \boxed{x^2+5x+6}$
sum $2+3$ product $2\cdot3$

② $(x-3)(x-2)$
$= x^2-2x-3x+6$
$= \boxed{x^2-5x+6}$
sum $(-2)+(-3)$ product $(-2)(-3)$

③ $(x+3)(x-2)$
$= x^2-2x+3x-6$
$= \boxed{x^2+x-6}$
sum $3+(-2)$ product $3(-2)$

④ $(x-3)(x+2)$
$= x^2+2x-3x-6$
$= \boxed{x^2-x-6}$
sum $(-3)+(2)$ product $(-3)(2)$

Chapter 6.2 Goal: Factor completely

⑤ x^2+5x+6 ⑥ x^2-5x+6 ⑦ x^2+x-6 ⑧ x^2-x-6

We know this basic structure:

$(x\quad)(x\quad)$

We need to determine #s and signs for each factor, knowing the sum and product of those two unknown numbers.

In each case, the numbers multiply to get the constant.

STEP 1: Look at the sign of the constant term (product) and determine if the signs of the factors are the same (both + or both −) or different (one +, one −)

(5) x^2+5x+6

↑ product is positive

⇒ signs are either both positive (as in (5)) or both negative (as in (6))

(6) x^2-5x+6

↑ product is positive

> If the constant term (product) is positive, the signs of the factors are the same.

Meanwhile...

(7) x^2+x-6

↑ product is negative

(8) x^2-x-6

↑ product is negative

⇒ signs must include one positive and one negative.

> If the constant term (product) is negative, the signs of the factors are different.

STEP 2: Look at the sign of the degree-1 term (sum) and determine

a) if same sign, both positive or both negative
b) if different signs, which number has larger magnitude.

(5) x^2+5x+6

mult to +6
add to +5
same signs → both positive

(6) x^2-5x+6

mult to +6
add to −5
same signs → negative.

> If the constant term is positive both numbers in factors will be same sign as the middle (or degree 1) term.

(7) x^2+x-6

mult to -6
add to $+1$
larger # is positive

(8) x^2-x-6

mult to -6
add to -1
larger # is negative

> If the constant term is negative, the larger number in the factor has the same sign as the middle (deg 1) term.

STEP 3: Make a list of factors, using signs discovered in steps 1 and 2.

(5) x^2+5x+6 — same sign, both positive
$1\times 6=6$
$2\times 3=6$.

(6) x^2-5x+6 — same sign, both neg
$(-1)\times(-6)=6$
$(-2)\times(-3)=6$

(7) x^2+x-6 — diff sign, larger is pos
$(-1)(6)=-6$
$(-2)(3)=-6$

(8) x^2-x-6 — diff signs, larger is neg
$1(-6)=-6$
$2(-3)=-6$.

STEP 4: Check the sum of each pair of factors in list from step 3.

(5) $1+6=7$ NO
$2+3=5$ ✓

(6) $(-1)+(-6)=-7$ NO
$(-2)+(-3)=-5$ ✓

(7) $(-1)+6=5$ NO
$(-2)+3=1$ YES

(8) $1+(-6)=-5$ NO
$2+(-3)=-1$ YES

STEP 5: If a pair of numbers multiplies and adds correctly, write the factors. If no pair of numbers works, this trinomial cannot be factored. (If no other factoring done, write PRIME.)

(5) $(x+2)(x+3)$

(6) $(x-2)(x-3)$

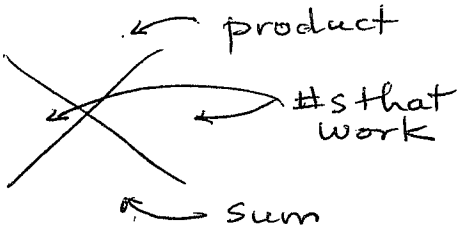
(7) $(x-2)(x+3)$

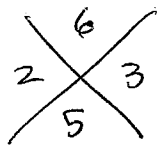
(8) $(x+2)(x-3)$.

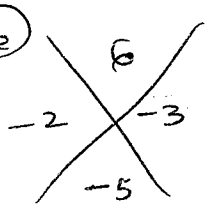
> Note: You can write the factors in either order.

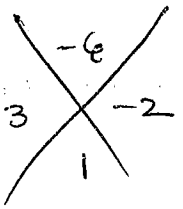
Optional Step 6: Check your work by FOIL!

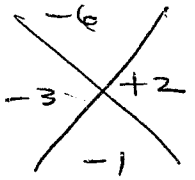
Some instructors (though not our current textbook) use the "magic X" as a way to write this down.



(5) 

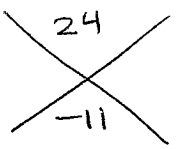
(6) 

(7) 

(8) 

Factor completely.

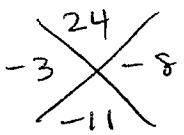
(9) $p^2 - 11p + 24$



product 24 positive ⇒ same signs

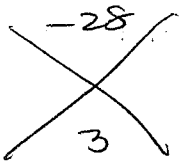
sum −11 negative ⇒ both negative

$(-1)+(-24) = -25$ no
$(-2)+(-12) = -14$ no
$(-3)+(-8) = -11$ yes
$(-4)+(-6) = -10$ no



$= \boxed{(p-3)(p-8)}$

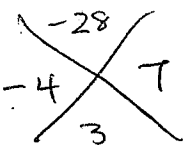
(10) $z^2 + 3z - 28$



product −28 neg. ⇒ diff signs

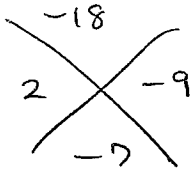
sum 3 pos ⇒ larger is positive.

$(-1)+(28) = 27$ no
$(-2)+(14) = 12$ no
$(-4)+7 = 3$ yes ✓



$= \boxed{(z-4)(z+7)}$

(11) $y^2 - 7y - 18$



product neg ⇒ diff signs

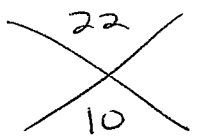
sum neg ⇒ larger is neg

$1+(-18) = -17$ no
$2+(-9) = -7$ yes ✓
$3+(-6) = -3$ no

$= \boxed{(y+2)(y-9)}$

Factor completely. or write prime.

(9) $y^2 + 10y + 22$



product positive ⇒ same sign

sum positive ⇒ both positive

$1 + 22 = 23$ no

$2 + 11 = 13$ no.

PRIME

CAUTION: Check for GCF!

(10) $2y^2 + 20y + 44$.

$= \boxed{2(y^2 + 10y + 22)}$

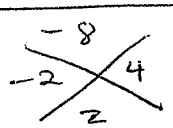
We do not write "prime" because we were able to factor out a GCF 2.

* Use PRIME only when no factoring of any type can be done

(11) $2w + w^2 - 8$

* Caution: If problem is not written in standard form, do that first.

$= w^2 + 2w - 8$



diff signs

Larger pos.

$-1 + 8 = 7$ no

$-2 + 4 = 2$ yes ✓

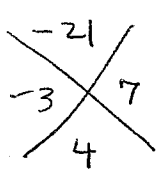
$= \boxed{(w-2)(w+4)}$

(12) $p^2 + 4pq - 21q^2$

structure of letters

$= (p \quad q)(p \quad q)$.

do numbers the same.



diff signs

larger pos.

$-1 + 21 = 20$ no.

$-3 + 7 = 4$ ✓

$= \boxed{(p - 3q)(p + 7q)}$

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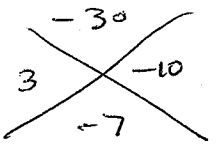
(13) $3u^3 - 21u^2 - 90u$

GCF $3u$

$= 3u\left(\frac{3u^3}{3u} - \frac{21u^2}{3u} - \frac{90u}{3u}\right)$

$= 3u(u^2 - 7u - 30)$

$= \boxed{3u(u+3)(u-10)}$



diff signs

larger is neg

$1 + (-30) = -29$
$2 + (-15) = -13$
$3 + (-10) = -7$ ✓
$5 + (-6) = -1$

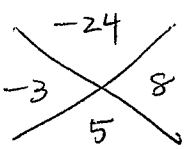
(14) $-w^2 - 5w + 24$

GCF -1

$= -\left(\frac{-w^2}{-1} - \frac{5w}{-1} + \frac{24}{-1}\right)$

$= -(w^2 + 5w - 24)$

$= \boxed{-(w-3)(w+8)}$



diff signs

larger pos.

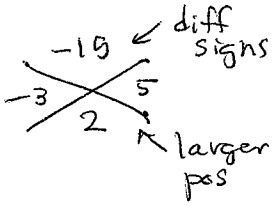
$-1 + (+24) = 23$
$-2 + 12 = 10$
$-3 + 8 = 5$ ✓
$-4 + 6 = 2$

(15) The area of a triangle is given by $\frac{1}{2}x^2 + x - \frac{15}{2}$ ($A = \frac{1}{2}bh$). Factor out $\frac{1}{2}$ then factor result to find binomials representing base and height.

factor out $\frac{1}{2}$: $\frac{1}{2}\left(\frac{\frac{1}{2}x^2}{\frac{1}{2}} + \frac{x}{\frac{1}{2}} - \frac{\frac{15}{2}}{\frac{1}{2}}\right)$

$= \frac{1}{2}(x^2 + 2x - 15)$

$= \boxed{\frac{1}{2}(x-3)(x+5)}$



diff signs

larger pos

$-1 + 15 = 14$
$-3 + 5 = 2$ ✓

(16) $-46x + 24x^2 - 2x^3$

step 1: write in standard form.

$= -2x^3 + 24x^2 - 46x$

step 2: factor out GCF, including negative.

$= -2x\left(\frac{-2x^3}{-2x} + \frac{24x^2}{-2x} - \frac{46x}{-2x}\right)$

$= -2x(x^2 - 12x + 23)$

step 3: factor trinomial

23 same sign

−12 both neg

$(-1) + (-23) = -24$ no

23 is prime, so no other factors exist.

$x^2 - 12x + 23$ cannot be factored.

$$\boxed{-2x(x^2 - 12x + 23)}$$

(17) $-16t^2 + 16t + 32$

GCF $= -16\left(\frac{-16t^2}{-16} + \frac{16t}{-16} + \frac{32}{-16}\right)$

$= -16(t^2 - t - 2)$

−2 ← different signs

−2 1

−1 ← larger is neg

$(-2) + (1) = -1$ ✓

$= \boxed{-16(t-2)(t+1)}$

Mixed Practice up to 6.2 p.8

Factor completely.

① $x^2 - ax + bx - ab$ — 4 terms ⇒ grouping

GCF x GCF b

$= x(x-a) + b(x-a)$

GCF $(x-a)$

$= \boxed{(x-a)(x+b)}$

② $-21x^3y - 14xy^2$

$= -7xy\left(\frac{-21x^3y}{-7xy} - \frac{14xy^2}{-7xy}\right)$ GCF $-7xy$

$= \boxed{-7xy(3x^2 + 2y)}$

③ $-36x + 18x^2 - 2x^3$

$= -2x^3 + 18x^2 - 36x$ standard form

$= -2x\left(\frac{-2x^3}{-2x} + \frac{18x^2}{-2x} - \frac{36x}{-2x}\right)$ GCF $-2x$

$= -2x(x^2 - 9x + 18)$

$= \boxed{-2x(x-3)(x-6)}$

18 ← same sign; -3×-6; -9 ← negative

④ $3x^3y - 6x^2y - 6xy$

$= 3xy\left(\frac{3x^3y}{3xy} - \frac{6x^2y}{3xy} - \frac{6xy}{3xy}\right)$ GCF

$= 3xy(x^2 - 2x - 2)$

$= \boxed{3xy(x^2-2x-2)}$

-2 ← diff signs; -2 ← largest neg

$1 + (-2) = -1$

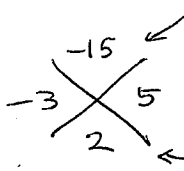
no other choice.

Mixed practice up to 6.2 p.9

(5) $m^2 + 2mn - 15n^2$

$\rightarrow (m \quad n)(m \quad n)$

$= \boxed{(m-3n)(m+5n)}$



← diff signs

← largest positive

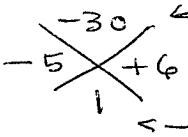
$-1 + (+15) = +14$

$-3 + (+5) = +2$ ✓

(6) $x^2 + xy - 30y^2$

$\rightarrow (x \quad y)(x \quad y)$

$= \boxed{(x-5y)(x+6y)}$



← diff signs

← largest pos.

$-1 + 30 = +29$

$-2 + 15 = 13$

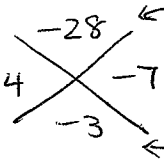
$-3 + 10 = 7$

$-5 + 6 = 1$ ✓

(7) $a^2 - 3ab - 28b^2$

$\rightarrow (a \quad b)(a \quad b)$

$= \boxed{(a+4b)(a-7b)}$



← diff signs

← largest neg

$1 + (-28) = -27$

$2 + (-14) = -12$

$4 + (-7) = -3$ ✓

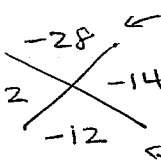
(8) $-x^3 + 12x^2 + 28x$

$= -x\left(\frac{-x^3}{-x} + \frac{12x^2}{-x} + \frac{28x}{-x}\right)$

$= -x(x^2 - 12x - 28)$

$= \boxed{-x(x+2)(x-14)}$

GCF



← diff signs

← largest neg

$1 + (-28) = -27$

$2 + (-14) = -12$ ✓